APPLICATION OF THE COMPUTATIONAL ALGORITHM OF THE MATRIX SWEEP METHOD

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ABSTRACT

The solution of inverse problems of thermal conductivity with a high-precision scheme is carried out by the method of approximate selection in the given article.

However, they try to use internal inverse problems of thermal conductivity to simplify calculations, which leads them to specific analytical expressions that are independent of the thermal effect, temperature field, and geometry of the sample. The solution of inverse problems of thermal conductivity with a high-precision scheme is carried out by the method of approximate selection.

The high-precision method is considered with the crank-Nicholson type difference scheme in this article. The crank-Nicholson type scheme is a parabolic differential equation of finite difference.

In particular, the advantage of the difference scheme is that the solution in the upper time layer is obtained immediately according to the values of the grid function in the lower time layer and it is without the solution of the system of linear algebraic equations (Satz), and also in it the solution becomes known (at k = 0, the values of the grid function are formed from the initial situation). But the same scheme has a significant disadvantage, since it is conditionally stable. On the other hand, the fuzzy difference scheme leads to the need to solve the SATs of a system of linear algebraic equations, but this scheme is absolutely stable.

The solution of the inverse problem of thermal conductivity with a high-precision circuit is defined as the limit of the solution group.For example, a well-known calculation algorithm of the Matrix method is used to solve a three-point boundary problem, the parameters of which satisfy the stability condition. Since there are inverse matrices, the necessary conditions for the stability of the Matrix method are met.

Keywords: heat conduction equations, increased accuracy method, regularized problems, ill-posed problems, approximation, stability, matrix sweep method, stability method.

INTRODUCTION

The solution of inverse problems of thermal conductivity with a high-precision scheme is carried out by the method of approximate selection in the given article.

However, they try to use internal inverse problems of thermal conductivity to simplify calculations, which leads them to specific analytical expressions that are independent of the thermal effect, temperature field, and geometry of the sample. The solution of inverse problems of thermal conductivity with a high-precision scheme is carried out by the method of approximate selection.

Let's consider the following mixed problem for the heat equation with reverse time flow

$$\frac{\partial \phi}{\partial t'} = \frac{\partial^2 \phi}{\partial x^2}, \qquad t' \in [0,T], \quad x \in [0;1] \tag{1}$$

$$\Phi\left(x,t'\right)\Big|_{t'} = T = u\left(x\right), \tag{2}$$

$$\varphi\left(0,t'\right) = \varphi\left(1,t'\right) = 0 \quad . \tag{3}$$

The method of improved accuracy- a difference scheme of the Crank-Nicolson type.is considered in the given article. [1] It is known that problem (1)-(3) is equivalent to the following problem:

$$\frac{\partial \phi}{\partial t} = -\frac{\partial^2 \phi}{\partial x^2}, \qquad t \in [(0,T], \quad x \in [0;1] \tag{4}$$

$$\Phi\left(x,t\right)\Big|_{t=0} = u\left(x\right), \tag{5}$$

$$\varphi\left(0,t\right) = \varphi\left(1,t\right) = 0\,. \tag{6}$$

The solution to problem (4)–(6) is defined as the limit of the solution family { φa (x, t)} of the following regularizable problems in the theory of ill-posed problems:

$$\frac{\partial \phi_a}{\partial t} = -\frac{\partial^2 \phi_a}{\partial x^2} - a\frac{\partial^4 \phi_a}{\partial x^4}, \qquad t \in [0,T], \quad x \in [0;1] \tag{7}$$

$$\varphi_a\left(x,0\right) = u\left(x\right),\ a > 0\,, \tag{8}$$

$$\varphi_a\left(0,t\right) = \varphi_a\left(1,t\right) = 0 \tag{9}$$

$$\frac{\partial^2 \phi_a}{\partial x^2}(0,t) = \frac{\partial^2 \phi_a}{\partial x^2}(1,t) = 0 \tag{10}$$

MATERIALS AND METHODS

Here *a* is a positive number, which is called the regularization parameter. In the works, the solutions (7) - (10) for each fixed value is classically correct and its solution φa(x,t) as α → 0 converges to the solution of the original ill-posed problem (1) - (3).

Approximation

Let $\varphi_a(x,t)=V(x,t)$ in a rectangle $\Pi:\{0\le x\le 1, 0\le t\le T\}$ introduce the following difference mesh:

$$\omega_{h\tau}=\{(x_k,t_n): x_k=kh,\ k=\overline{0,N};\ h=1/N; t_n=n\tau,\ n=\overline{0,M};\ \tau=T/M\}.$$

In internal nodes (x_k,t_n) $(k=\overline{1,N-1};\ n=\overline{1,M})$ difference mesh$\omega_{h\tau}$зthe solutions (7)–(10) is approximated by the difference problem

$$\frac{V_k^{n+1}-V_k^n}{\tau}=\lambda^{(\alpha)}\frac{V_k^{n+1}+V_k^n}{2}\quad (k=\overline{1,N-1};\quad n=\overline{0,M-1}) \tag{11}$$

$$V_0^n=(u(x_1),u(x_2),\dots,u(x_n))^T=\bar{u}, \tag{12}$$

$$V_0^n=V_N^n=0,\ V_{-1}^n=-V_0^n,\ V_{N+1}^n=-V_{N-1}^n\quad (n=\overline{0,M}). \tag{13}$$

Here the difference operator A_α^0 isdefined as

$$A_\alpha W_k=(\lambda_{xx}-\alpha(\lambda_{xx})^2)W_k\ ,\ \lambda_{xx}W_k=-\frac{W_{k-1}-2W_k+W_{k+1}}{h^2}.$$

Let us expand the mesh functions in a Taylor series $V_{k-2}^{n+1}, V_{k-1}^{n+1}, V_{k+1}^{n+1}, V_{k+2}^{n+1}$ in the neighborhood of point (x_k, t_n) [1,2].

Difference solutions (11) – (13) approximates solutions (7) – (10) with order 0 $\left(\tau^2+h^2\right)$ in the class of solutions that have continuous partial derivatives up to the third order in t and up to the sixth order in x.

RESULTS AND DISCUSSION

Solvability

To prove the unique solvability of the difference solution (11) -(13), we represent it in the following form:

$$\left(E-\tau A_a/2\right)V^{n+1}=\left(E+\tau A_a/2\right)V^n, \tag{14}$$

$$V^0=\bar{u}, \tag{15}$$

where

$$V^{n+1}=\left(V_1^{n+1},\ V_2^{n+1},\dots,V_{N-1}^{n+1}\right),$$
$$V^n=\left(V_1^n,V_2^n,\dots,V_{N-1}^n\right),$$

$A_a = \{a_{ij}^{(a)}\}_1^{n-1}$ - symmetric square five-diagonal matrix of the same order with entries $a_{ij}^{(a)}$:

$$a_{11} = a_{N-1N-1} = -5a/h^4 + 1/h^2$$
$$a_{ij}^{(a)} = \begin{cases} 4a/h^4 - 1/h^2, \text{ if } |\text{i-j}| = 1; \\ -a/h^4, \quad \text{if } |\text{i-j}| = 2; \\ -6a/h^4 + 2/h^2, \text{ if i} = 0, \quad (\text{i,j} = 2,3,...,\text{n-2}). \end{cases}$$

Proximately the same as the symmetric matrix B_a=E- $\tau A_a/2$ has the following eigenvalues:

$$\lambda_B(B_a) = 1 - \frac{2\tau}{h^2}\sin^2\frac{s\pi h}{2}\left(1 - \frac{4a}{h^2}\sin^2\frac{s\pi h}{2}\right), \text{s} = 1,2,..., \text{N - 1}.$$

We choose the regularization parameter *a* and the difference scheme steps h and τ from the condition:

$$h^2 \le 8a\,,\ \tau \le 16a \qquad (16)$$

then all $\lambda_S(B_a)$ $(s = 1, 2, ..., N-1)$ will be positive, and the matrix B_a is improper, since

$$\min_s \lambda_s(B_a) = \lambda_{s0} = 1 - \frac{s\pi h}{2}\left(1 - \frac{4a}{h^2}\sin^2\frac{s\pi h}{2}\right), \text{ s} = 1,2,..., \text{N - 1}.$$

Therefore, solution (3.3.14) - (3.3.15) is uniquely solvable.

Stability

We represent the matrix A_a in the shape of a sum

$$A_a = A_h - aA_h\,, \qquad (17)$$

where $A_h = \{a_{ij}^{(h)}\}_1^{N-1}$ - square matrix with entries:

$$a_{ij}^{(a)} = 1/h^2 \begin{cases} 0, \quad \text{at } |\text{i-j}| > 2; \\ -1, \quad \text{at } |\text{i-j}| = 1; \\ 2, \quad \text{at } |\text{i-j}|. \end{cases}$$

It is known that the matrix A_h - is definedpositively. [3,4] Taking into account the representation (17), we rewrite the difference equation (14) in the form of:

$$\frac{V^{n+1}-V^{n}}{\tau}=A_h\frac{V^{n+1}-V^{n}}{2}-aA_h^2\frac{V^{n+1}-V^{n}}{2}.$$

The last equation is scalar multiplied by the vector $W^{n+1/2} = V^{n+1} + V^{n}$:

$$\frac{1}{\tau}\left(\left\|V^{n+1}\right\|^2-\left\|V^{n}\right\|^2\right)=\frac{1}{2}\left(A_hW^{n+1/2},W^{n+1/2}\right)-\frac{a}{2}\left(A_hW^{n+1/2},\mathrm{A_h}W^{n+1/2}\right).$$

From this equation we obtain the following congruence:

$$\left\|V^{n+1}\right\|^2+\frac{a\tau}{2}\left\|A_hW^{n+1/2}\right\|^2=\left\|V^{n}\right\|^2+\frac{\tau}{2}\left(A_hW^{n+1/2},\ \mathrm{W}^{n+1/2}\right). \qquad (18)$$

In the well-known young inequality

$$\left|\left(A_hW,W\right)\right|\le\frac{\varepsilon}{2}\left\|A_hW\right\|^2+\frac{1}{2\varepsilon}\left\|W\right\|^2, \qquad (\varepsilon>0)$$

Let us assume that $\varepsilon = 2a$ *:*

$$\left|\left(A_hW,W\right)\right|\le a\left\|A_hW\right\|^2+\frac{1}{4a}\left\|W\right\|^2,$$

We rewrite congruence (18) in the form:

$$\left\|V^{n+1}\right\|^2\le\left\|V^{n}\right\|^2+\frac{\tau}{8a}\left\|V^{n+1}+V^{n}\right\|^2.$$

From the last congruence we determine:

$$\left\|V^{n+1}\right\|^2\le\frac{1+\tau/4a}{1-\tau/4a}\left\|V^{n}\right\|^2=R\left\|V^{n}\right\|^2,$$

where

$$R=\left(1+\tau/4a\right)\cdot\left(1+\tau/4a\right)^{-1}.$$

If we assume $n=0,1,\dots,M-1$, *(* $n\tau\le T$ *)*, then under condition (16) it is easy to establish that for any *n* we have the estimate

$$\left\|V^{n}\right\|\le\exp\left(T/\left(4aC\right)\left\|V^{0}\right\|\right)=\exp\left(T/\left(4aC\right)\right)\left\|\bar{u}\right\|,\text{where}\ C=1-\tau/4a.$$

The last estimate characterizes the stability of the difference scheme (14)–(15).

Solution method

In our case, the matrix sweep method is convenient.
To do this, we rewrite problem (14)–(15) in a vector form.

$$B_0\overline{V}_0^{n+1} + C_0\overline{V}_1^{n+1} = \overline{F}_0^{n},$$
$$A\overline{V}_{k-1}^{n+1} + B\overline{V}_k^{n+1} + C\overline{V}_{k+1}^{n+1} = \overline{F}_0^{n} \qquad (k = 1, 2, ..., N-1),$$
$$A_n V_{N-1}^{n+1} + B_N\overline{V}_N^{n+1} = \overline{F}_N^{n},$$

where matrixes A, B, C, B_0, C_0, A_N, B_N and vectors F_0^n, F_k^n, F_N^n are expressed in terms of numbers:

$a = \frac{2\tau}{2h^4}$, $b = 1 - \frac{\tau}{2}\left(\frac{4a}{h^4} - \frac{1}{h^2}\right)$, $C = 1 - \frac{\tau}{2}\left(-\frac{6a}{h^4} + \frac{1}{h^2}\right)$, as well as boundary conditions (15).

CONCLUSION

The well-known computational algorithm of the matrix sweep method is used to solve the three-point boundary value problem. Parameters τ, h and a are chosen to satisfy the stability condition [5]. Since reciprocal matrices $B_0^{-1}, B^{-1}, B_N^{-1}$, exist, the necessary conditions for the stability of the matrix sweep method are satisfied.

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